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Application of Probabilistic Fracture Mechanics to Prognosis of Aircraft Engine Components

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It is generally accepted that traditional logistics functions including periodic nondestructive inspections and planned maintenance increase the reliability and readiness of turbine engines. Nevertheless, further significant enhancements in reliability and readiness are believed to be possible through the implementation of a prognosis system based on online monitoring and interpretation of critical engine operating parameters and conditions to diagnose potential problems and forecast readiness. An approach is presented for improving probabilistic life prediction estimates through the application of prognosis methods. Actual F-16/F100 usage data from flight data recorders were interfaced with a probabilistic life prediction code to quantify the influence of usage on the probability of fracture of an idealized titanium compressor disk. For the example cases considered, it is shown that usage variability leads to about 6 × variability in life and from 10 × to 100 × variability in the probability of fracture. The results suggest that variability in usage could provide a basis for selectively extending the life of aircraft engines.

Nomenclature

$a_{\max}$	=	maximum defect area
$a_{\min}$	=	minimum defect area
$D(a)$	=	expected number of defects of area a
da/dN	=	crack growth rate
F_{i+p}	=	probability of fracture associated with fatigue
F_p	=	probability of fracture associated with manufacturing-related anomalies
$F_{X_1}(a)$	=	defect size cumulative distribution
$f(a)$	=	probability density function associated with defect of area a
f_{N_i}	=	probability density of N_i
$g(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{N})$	=	limit state associated with crack growth life
K_c	=	fracture toughness
$K(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{N})$	=	stress intensity factor
l	=	stochastic crack growth life
l_{model}	=	deterministic crack growth life predicted using established equations and algorithms
m	=	crack growth rate exponent
$\boldsymbol{N}$	=	vector of variables related to crack initiation and propagation
N_i, n_i	=	number of flights associated with crack initiation
N_{insp}	=	number of flights associated with inspection
N_p	=	number of flights associated with crack propagation
P_{det}	=	probability of detecting defect from population of defects
POD (a)	=	probability of detection (POD) of defect with area greater than a
s	=	applied stress
s_{FEM}	=	finite element method (FEM) analysis stress result at initial defect location
$\boldsymbol{X}$	=	vector of variables unrelated to inspections
X_1	=	initial defect area
X_2	=	stress scatter factor
X_3	=	crack growth life scatter factor
$\boldsymbol{Y}$	=	vector of variables related to inspections
$\Delta\sigma_a$	=	applied stress range

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Introduction

A WIDE variety of hardware and software tools is available for prognosis. Two fundamentally different approaches can be used to complete the process: 1) traditional health monitoring with signal-processing-based diagnosis and prognosis and 2) probabilistic life prediction. The traditional health monitoring approach relies on data interrogation methods such as pattern recognition, neural networks, expert systems, and fuzzy logic to diagnose and treat problems. Although this approach is computationally efficient (making it well suited to onboard prognosis), it requires some form of calibration or training during the diagnosis and prognosis steps to provide meaningful information for the final decision. In contrast, the probabilistic life prediction approach uses a physics-based model that includes detailed information from finite element analysis and damage accumulation algorithms. Because this approach can require significant computations, it is presently more appropriate for ground-based analysis and interpretation of downloaded data. The fundamental goal of both approaches is to provide information for making better-informed decisions, and so in many respects the aforementioned tools can be viewed as being complementary.

Traditional Health Monitoring Approach

Traditional health monitoring based on gas path analysis (GPA)[1,2] introduces a number of errors into the life prediction of gas-turbine engines. The amplitude of background noise is often the same order of magnitude as that of an engine fault, making it difficult even for a human expert to isolate faults based on GPA data alone. Bias of measured values occurs frequently, for example, improper signal grounding.[3] Because a large number of sensors is often required to measure the relatively large number of parameters typically associated with complete fault diagnostics, the cumulative bias can be significant. Moreover, the nonlinear relationship between measured parameters and performance introduces additional complexity into GPA models.[4–6]

A number of estimation methods have been developed to address the noise and bias associated with sensor measurements, such as Kalman filtering, forgetting factor, and various least-squares techniques.[7] The Kalman filter (KF) has been used extensively to study the effects of long-term deterioration.[8] Some additional statistical assumptions are required to apply the KF to gas turbines, for example, specification of probability density functions associated with prior distributions.[9] Furthermore, because it is limited to linear processes, the KF cannot be used to capture the nonlinear behavior typically associated with gas-turbine engines.

Neural networks (NN) are a practical alternative for addressing noise measurement bias. NN can be used to identify relationships among measured and performance parameters using a series of interconnected processing elements designed to simulate the biological learning process. Several methods can be used to train NN to detect faults, for example, unsupervised or reinforcement learning. Backpropagation learning is the most popular method when historical data are available.[10] Unfortunately, backpropagation learning can be computationally intensive, particularly for the complex structure of nets required for isolation of faults in gas-turbine engines.[8,9] A further disadvantage of NN is that because they are typically trained at steady-state conditions, they do not consider transient conditions that usually have a higher associated variability. Because engine faults can often occur during nonsteady-state conditions, this can be a significant disadvantage.

Several hybrid health-monitoring techniques have been developed to address some of the issues associated with application of NN to gas-turbine engine prognosis. For example, in Ref. 11, a combined technique of NN and expert systems is used to predict the behavior of the CFM56-3 engine for Boeing 737 commercial aircraft. A method is described in Ref. 4 that combines NN with fuzzy logic to account for transient conditions not covered by NN. In some instances, NN are replaced altogether with other predictive methods such as adaptive regression[7] or genetic algorithms.[9] Similar to NN, these methods require some form of calibration. Additional assumptions regarding the statistical behavior of input variables are often required as well.

Probabilistic Life Prediction

Probabilistic methodologies are being increasingly used to the address the uncertainties associated with fracture-mechanics-based life prediction of aircraft engine components. These methodologies provide the capability to quantify the risk of fracture associated with rare, life limiting events such as inherent material and manufacturing defects that occasionally occur during processing. Over the past several years, a probabilistic methodology has been under development for crack growth life prediction of rotors and disk in commercial aircraft gas-turbine engines.[12–15] The methodology was recommended by the Federal Aviation Administration[16] to address the occurrence of relatively rare metallurgical defects that can lead to uncontained engine failures.[17] It has been implemented in a probabilistic damage tolerance computer program called DARWIN.[18–23]

For a probabilistic assessment of crack growth life, statistical descriptions and relationships among a number of random variables are required. Although a number of factors influence the crack propagation life of aircraft turbine rotors and disks, the most influential ones are related to the size and location of initial defects, material property scatter, and applied stress.[14,18,24] Detailed defect data are typically not available for most materials. However, for titanium materials with hard alpha inclusions, anomaly distributions are available for use in probabilistic life assessment.[25] Scatter in the crack growth rate due to material variability can be readily quantified using well-established test methods. On the other hand, crack growth rate $\mathrm{d}a/\mathrm{d}N$ is highly dependent on the applied stress range $\Delta\sigma_a$, that is, in the power law regime, $\mathrm{d}a/\mathrm{d}N = f(\Delta\sigma_a)^m$, where m is typically between 3 and 4 for titanium alloys. Because the applied stress is directly related to the usage history, its associated variability is often difficult to predict a priori without some knowledge of the intended use.

The application of prognosis methods to probabilistic life prediction can significantly improve risk/performance estimates by providing key information regarding usage.[26] Using data from sensors to quantify the stresses associated with usage and combining them with probabilistic life prediction can lead to improved decisions regarding aircraft readiness.

Probabilistic Modeling

The crack growth life of components that can lead to uncontained engine failures (e.g., rotors and disks) is considered, where the limit state is defined as[14]

$$g(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{N}) = K_C - K(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{N}) \leq 0 \tag{1}$$

where g $(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{N})$ is dependent on random variable vectors $\boldsymbol{X}$ (variables unrelated to inspections), $\boldsymbol{Y}$ (variables related to inspections), and $\boldsymbol{N}$ (number of flights to initiate and propagate cracks):

$$g(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{N}) = g(X_1, \ldots, X_n; Y_1, \ldots, Y_m; N_i, N_p) \tag{2}$$

When the structural integrity of a titanium rotor disk containing hard alpha anomalies is considered, the potential $\boldsymbol{X}$ random variables include defect size and location, stress, and material properties. The time and effectiveness of the inspections are among the potential $\boldsymbol{Y}$ random variables. The effectiveness of an inspection can be characterized by its probability of detection (POD) distribution.

Three $\boldsymbol{X}$ random variables are considered: initial defect area X_1, stress scatter factor X_2, and crack growth life scatter factor X_3. Inherent material anomalies are modeled using established data for titanium materials[16] converted to a cumulative distribution function (CDF) format[14,18]:

$$F_{X_1}(a) = \left\{ \begin{array}{ll} 0, & a < a_{\min} \\ 1 - \dfrac{D(a) - D(a_{\max})}{D(a_{\min}) - D(a_{\max})}, & a_{\min} \leq a \leq a_{\max} \\ 1, & a > a_{\max} \end{array} \right\} \tag{3}$$

Applied stress is modeled as the product of finite element method (FEM) analysis results and stress scatter factor X_2,

$$s = X_2 \cdot s_{\mathrm{FEM}} \tag{4}$$

Similarly, stochastic crack growth life is defined as

$$l = X_3 \cdot l_{\mathrm{model}} \tag{5}$$

where l_{model} is the predicted fatigue life based on established fatigue crack growth equations and algorithms.

Two $\boldsymbol{Y}$ random variables are considered: N_{insp}, the number of cycles associated with inspection, and P_{det}, the probability of detecting a defect from a population of defects. P_{det} is defined as

$$P_{\mathrm{det}} = \int_0^\infty \mathrm{POD}(a) \cdot f(a)\,\mathrm{d}a \tag{6}$$

Anomalies introduced during manufacturing (e.g., inherent material defects and surface damage) are assumed to appear as cracks during the first cycle of applied load (i.e., $N_i = 0$ flights). The probability of fracture associated with these anomalies is given by

$$F_p = P[g(\boldsymbol{X}, \boldsymbol{Y}, N_p) \leq 0] \tag{7}$$

Complete details regarding the stress intensity factor solution are provided in Ref. 18. A summary of the random variables associated with Eq. (1) is provided in Refs. 14 and 18.

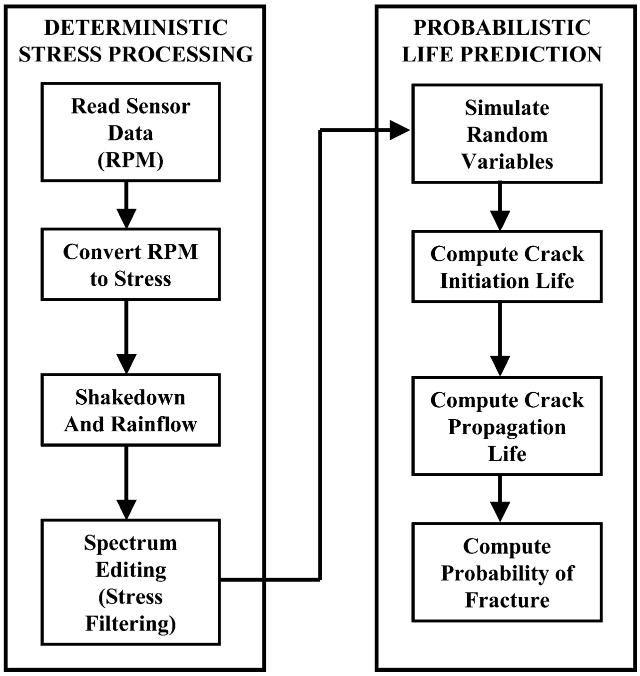

Fig. 1 Approach for introducing sensor data into probabilistic life prediction computations.

If fatigue is also considered, the probability of fracture is also dependent on the number of flights N_i required to initiate a crack:

$$F_{i+p} = \int F_p(n_i) f_{N_i}(n_i)\, \mathrm{d}n_i \tag{8}$$

The crack initiation life N_i was modeled as the number of flights required to initiate a detectable crack length of 0.076 cm (0.030 in.). N_i was estimated using values obtained from a curve fit of strain-life data. [A modified form of the well-known Smith–Watson–Topper model (see Ref. 27) was used to account for mean stress effects.] The crack propagation life N_p was modeled as the number of flights associated with violation of the fracture limit state [Eq. (1)].

A summary of the approach for incorporating sensor data into a probabilistic fracture mechanics framework for prognosis-based life prediction is shown in Fig. 1. Raw revolutions per minute usage data (obtained from flight data recorders used for actual F-16/F100 missions) are converted to a number of ordered stress values based on empirical results at several turbine disk locations. Shakedown is used to identify residual stresses associated with stress values that are above the material yield stress. The residual stresses are applied as necessary, and the stress pairs are reordered using an established rainflow cycle counting algorithm.[27] The stress pairs are further processed using a stress-range-based spectrum editing algorithm to remove nondamaging pairs and are applied as deterministic input to the probabilistic life prediction module to predict probability of fracture vs cycles.

Probabilistic Life Prediction for Actual Usage Histories

To demonstrate the feasibility of the probabilistic life prediction approach for prognosis, actual F-16/F100 usage data from flight data recorders were successfully interfaced with a prognosis-enhanced version of DARWIN for a generic titanium alloy compressor disk shown in Fig. 2. Several crack locations were considered; however, this study focuses on a surface crack initiating from a bolt hole with applied stresses in the hoop direction (Fig. 2). The following failure modes were included in the analysis: crack initiation, crack propagation, and combined crack initiation plus propagation. Although a number of factors were considered, the primary focus of this paper is on the influence of the various usages on the probability of failure of a typical titanium rotor disk. To demonstrate clearly the maximum influence of usage, it was assumed that the engine was

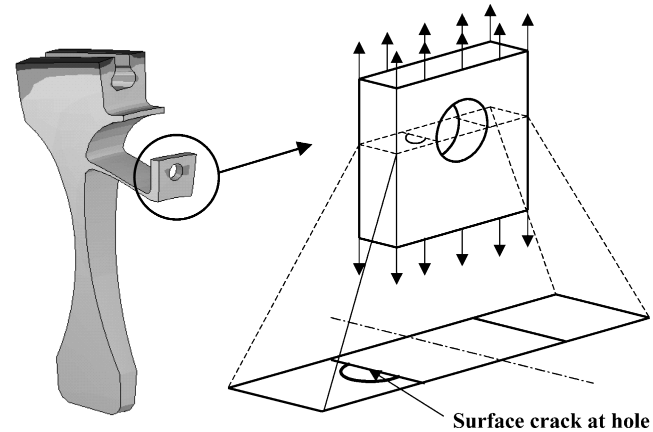

Fig. 2 Application of probabilistic prognosis approach to life prediction of aircraft gas-turbine engine compressor disk.

repeatedly subjected to the same mission profile with no mission mixing. Typical usage data for a variety of military aircraft engine (F-16/F100) mission profiles were considered. In general, the mission duration and speed values vary significantly from mission to mission. Mission profiles associated with some of the most severe and least severe missions (in terms of damage) are shown in Fig. 3.

Failure probabilities (computed using the prognosis enhanced DARWIN computer code) are shown in Fig. 4 for five typical missions. In Fig. 4, failure probabilities are based on the crack initiation plus propagation failure limit state, and failure probability values are normalized to the most damaging mission. Note that, for a given target failure probability, the number of flights to failure for the least damaging mission (air–ground weapons delivery) is up to six times greater than the most damaging mission (air–air weapons delivery). Alternatively, at a specified number of flights, the failure probability can vary by one to two orders of magnitude or more depending on the mission profile used. These differences can become more pronounced if intramission variability, that is, variability within a specific mission, is also taken into account.

Intramission variability is shown in Figs. 5 and 6 for missions with relatively high variability (air–air weapons delivery) and low variability (combat air patrol), respectively. Although the mean lives are similar, the dispersion about the mean lives is considerably different for the two mission types, resulting in significantly different failure probabilities. This concept is shown in Fig. 7, in which the lognormal crack growth life probability density functions (based on deterministic life values associated with repeated application of specific missions) are shown for these two mission profiles, illustrating the intramission variability. By comparison of the left tails of the distributions, it can be observed that the extreme values associated with the air–air weapons delivery mission are greater than that of the combat air patrol mission, consistent with the results shown in Fig. 4. This further illustrates the mission-specific influence of intramission variability on life for the idealized situation in which the engine is repeatedly subjected to the same mission profile.

The results in Figs. 3–7 demonstrate that significant variability occurs in engine usage, which translates into significant variability in component life/reliability. Because information was not available to the authors on how the actual usage employed in these simulations compares to the usage assumed in design, or in engine structural integrity program calculations, it is unclear whether the actual usage employed here is conservative or nonconservative compared to the usage assumed when setting life limits on engine components. Current practice in the engine community is to use typical usage in design and structural integrity analyses, although it is not uncommon to update this assumption as additional usage data are periodically acquired. Simplified rules for mission mixing represent another assumption in these analyses and, therefore, a source of additional variability and possible bias.

When the variability in computed life demonstrated here is considered, there would appear to be opportunities to extend the life

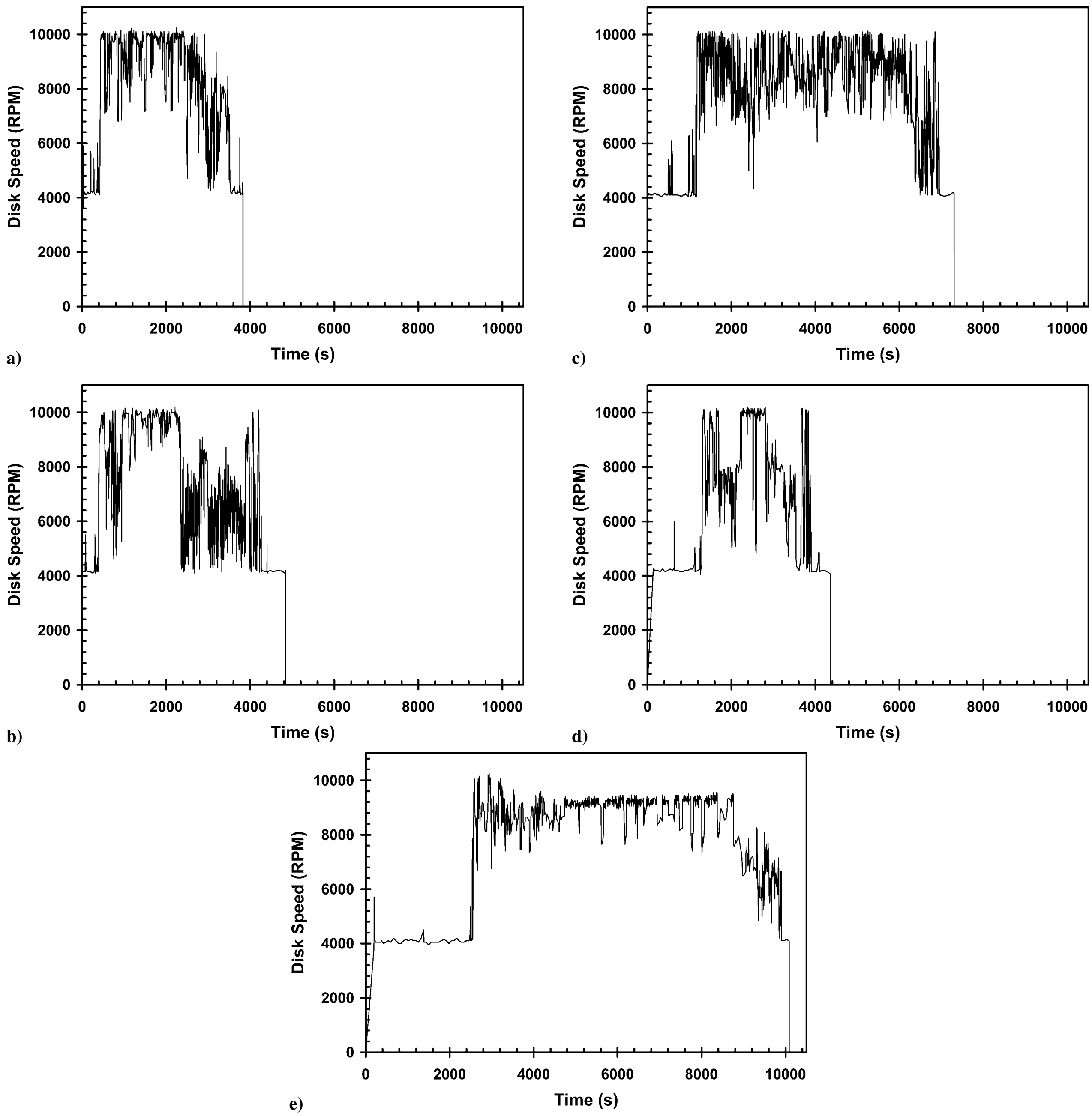

Fig. 3 Actual usage data for several typical mission profiles: a) air combat tactics, b) combat air patrol, c) air–ground weapons delivery, d) air–air weapons delivery, and e) instrument/ferry.

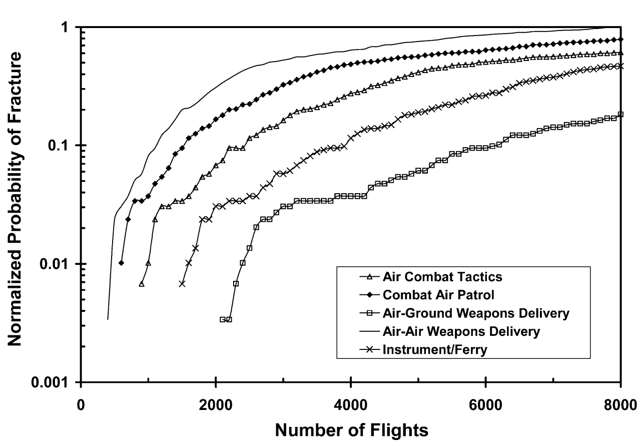

Fig. 4 Influence of mission type (intermission variability) on normalized failure probability.

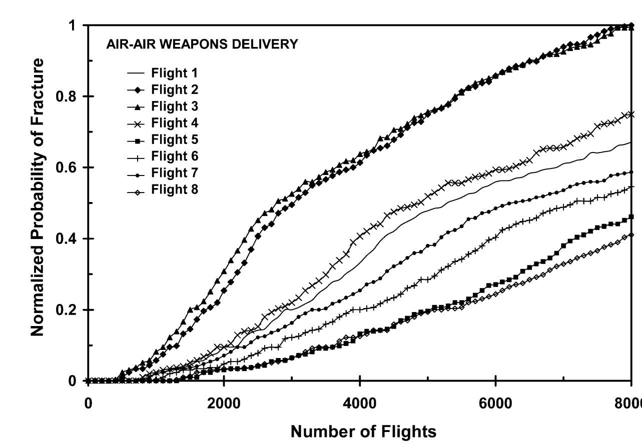

Fig. 5 Intramission usage variability associated with air–air weapons delivery mission.

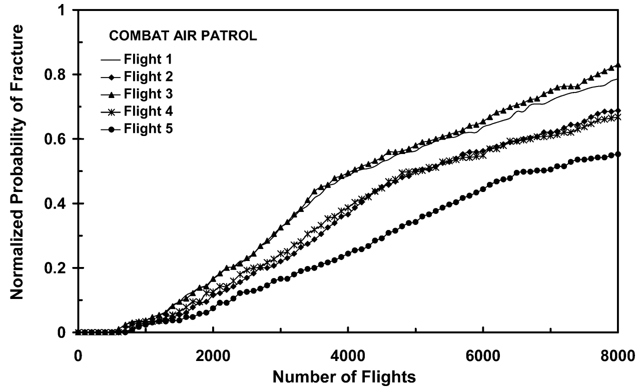

Fig. 6 Intramission usage variability associated with combat air patrol mission.

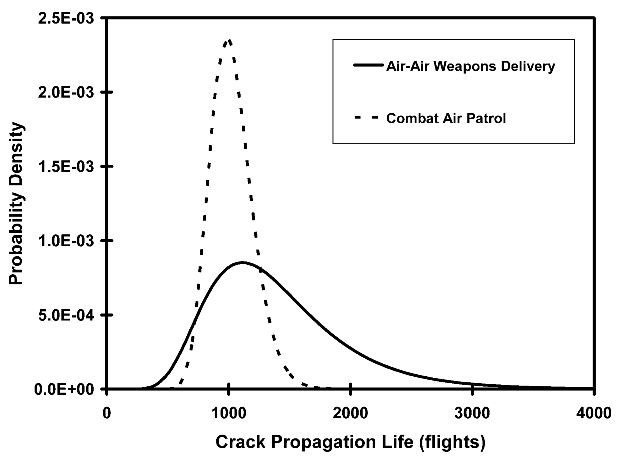

Fig. 7 Comparison of crack propagation life density functions for air–air weapons delivery and combat air patrol mission profiles.

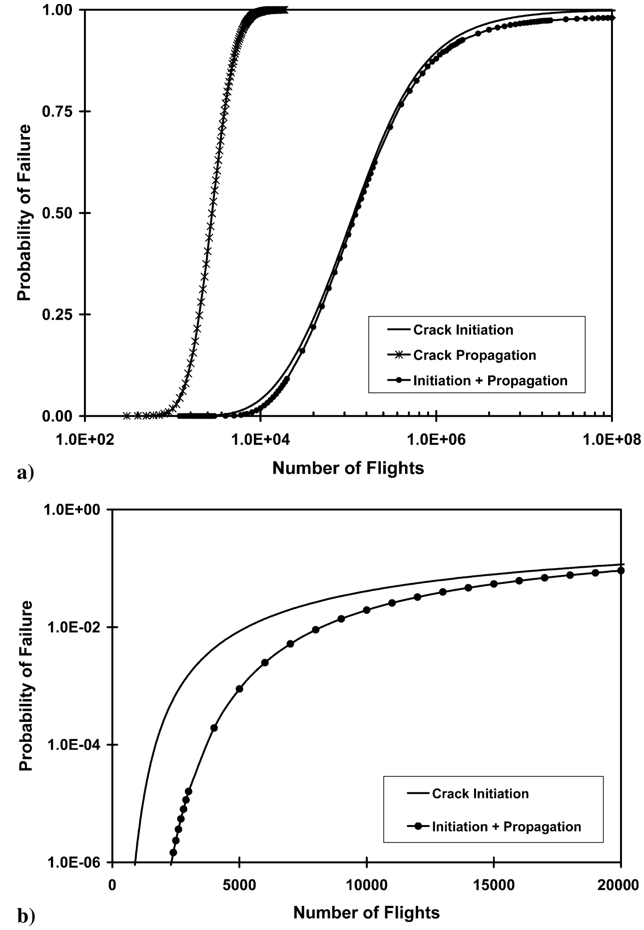

Fig. 8 Relative contributions of crack initiation and propagation to total life: a) entire CDF and b) partial CDF focusing on lower tail region.

of at least some fraction of the total component population if usage and damage could be tracked on the component level, at least for the most critical components. The benefit could be a significant savings in component replacement costs. A better estimate of the extent of this life extension benefit could be obtained by comparing the difference between the typical usage assumed in setting the safe life limit and the mean value of the actual usage used in the current computations. If the assumed typical usage is a conservative estimate of the actual usage, then the life extension benefits would be greater than those suggested by the present analysis of usage variability. Unfortunately, information on the assumed typical usage is not available to the authors.

Although the relatively large variability in the actual usage and the resulting 6× variability in the computed life suggests that significant life extension benefits could be accrued, it is important to recognize that the current computations were somewhat idealized. Specifically, the computations performed herein assumed that a given engine/component experiences the same usage throughout its life, that is, exactly the same mission was repeatedly flown to failure. Mission mixing would tend to drive the usage on any given component toward a mean value, thereby decreasing the observed variability in computed life shown in Figs. 4–7.

Usage variability also has consequences on prognosis of future reliability for mission planning purposes. The significant inter- and intramission variability presents challenges for forward projections of the consequences of future usage changes. However, we believe that accurate statistical models can be formulated, but would likely need more usage data to ensure robustness. Such modeling would be useful to examine the consequences of significant changes in mission usage on mission planning, as well as reliability and sustainability (inspection, maintenance, and repair).

If probability of failure indices were available for specific engines/components, they could be used to optimize mission planning by selecting aircraft with the lowest risk (lowest P_f). To accomplish such mission planning in the field, the usage of fracture critical engine components would need to be tracked over their lifetime and probabilistic computation updated periodically. If such a system were web based, data and resulting computations could be efficiently shared between field operations and depots.

In Fig. 8, the failure probability curve is separated into components associated with crack initiation (probability that the number of flights is greater than N_i) and crack growth (F_p) and is compared to the failure probability associated with the total (initiation plus propagation) life (F_{i+p}). Note that, for Fig. 8, the mean life, that is, $P_f = 0.5$, is dominated by crack initiation, whereas in the left tail of the curves (at low probabilities of failure, $P_f = 10^{-3}$; Fig. 8b), the fatigue life consists of equal contributions from crack initiation and crack growth. This occurs because scatter in initiation lives is considerably greater than scatter in propagation lives. Although the relative contributions of the mean crack initiation and propagation life values may vary for different materials, it is essential to consider their relative influences on the probability of failure.

Summary

Usage data from flight data recorders were successfully interfaced with a probabilistic life prediction code and employed to identify the sensitivity of component life and probability of failure to usage variability. Analysis of actual F-16/F100 flight recorder data indicates significant inter- and intramission usage variability. For the example cases examined (idealized situation in which the engine is repeatedly subjected to the same mission profile with no mission mixing), this variability corresponded to about 6× variability in life and from 10× to 100× variability in probability of failure at a given life. Because safe life limits within the engine community are generally performed with typical mission usage and simple rules for estimating mission mixing, these results suggest that variability in usage could provide a basis for selectively extending the life of at least a portion of the fleet.

Capitalizing on usage variability to extend the life of new engines would require usage to be tracked on individual engine components. Capitalizing on usage variability to extend the life of legacy engines would require additional statistical modeling to simulate past performance to overcome limitations in the ability to trace prior mission history for a given engine/component.

Forecasting future reliability of engines for mission planning is feasible with the methods demonstrated here, but would require the development of stochastic models to describe variability in mission usage and mission mixing.

The results obtained in the study serve to demonstrate the utility of integrating probabilistic damage assessments into the prognosis process, for both enhanced mission planning and sustainment; several means of improving computational efficiency were also identified that will likely facilitate the transition of such computations from on-ground to onboard.

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